

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. (a) Let $\{A_n\}, n=1, 2, \dots$ be a non decreasing sequence of events and let A be their alternative. Then we have prove that $\lim_{n \rightarrow \infty} P(A_n) = P(A)$.
- (b) Show that : If events A and B satisfy the condition $A \subset B$, then $P(A) \leq P(B)$.
17. Derive the Gammr distributions.
18. Derive the characteristic function and moments.
19. Derive Cauchy and laplace distributions.
20. Prove that : If the sequence $\{F_n(x)\}(n=1, 2, \dots)$ of distribution functions is convergent to the distribution function $F(x)$, then the corresponding sequence of characteristic fractions $\{\phi_n(t)\}$ convergent it every point $t(-\infty < t < \infty)$ to the function $\phi(t)$ which is the characteristics function of the limit distribution function $F(x)$ and the convergence to $\phi(t)$ is uniform with respect to t every finite interval on the t -axis.

NOVEMBER/DECEMBER 2025

23PMA32 — PROBABILITY THEORY

Time : Three hours

Maximum : 75 marks



SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. What is random event?
2. Define probability function.
3. What about distribution function?
4. What is standard deviation?
5. Define expected value of random variable.
6. What do you mean by characteristic function?
7. Define one point distribution.
8. Explain stochastic convergence.
9. What is normal distribution?
10. Write the formula of characteristic function $\phi_n(t) = ?$, $\phi(t) = ?$.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Explain marginal distribution.

Or

- (b) If events A and B satisfy the condition $A \subset B$, prove that $P(A) \leq P(B)$.

12. (a) Show that let the random variable x take on the values $k=0, 1, 2, \dots$ and let

$P(x=k) = \frac{\lambda^k}{k!} e^{-\lambda}$, where $\lambda > 0$ is a certain positive constant.

Or

- (b) Prove that if a random variable y can take on only non-negative values and has expected value $E(Y)$, then for an arbitrary positive number K .

$$P(Y \geq k) \leq \frac{E(Y)}{K}.$$

13. (a) Show that the random variable x has a binomial distribution, that is,

$$P_k = \binom{n}{k} p^k (1-p)^{n-k} \quad (k=0, 1, 2, \dots, n).$$

Or

- (b) Define probability generating function. Prove that the random variable X has a poisson distribution, (i.e) $P_k = e^{-\lambda} \frac{\lambda^k}{k!}$ ($k=0, 1, 2, \dots$).

14. (a) Derive poisson distribution.

Or

- (b) Discuss about one point and two point distribution.

15. (a) Prove that the sequence of random variable $\{x_n\}$ given by equation is stochastically convergent to 0, that is, for any $\epsilon > 0$ we have $\lim_{n \rightarrow \infty} P(|x_n| > \epsilon) = 0$.

Or

- (b) Derive a the Lindeberg-Levy theorem.